

# Chapter 4. Variability

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2021-09-27

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1. Introduction of Variability
2. Variance and Standard Deviation for a Population
3. Variance and Standard Deviation for a Sample
4. More about Variance and Standard Deviation
5. Range and interquartile range

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# Introduction of Variability

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- In simple terms, if the scores in a distribution are all the same, then there is no variability.

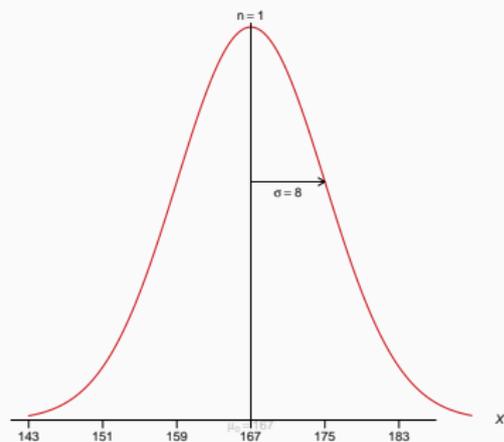
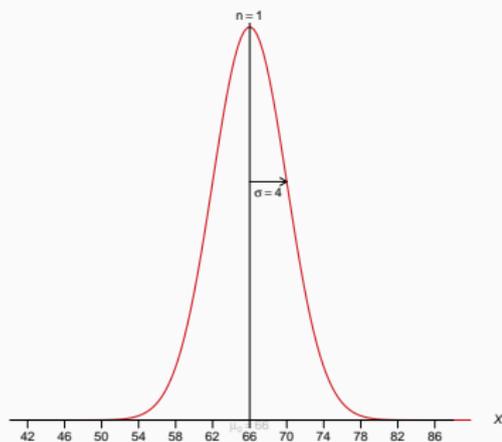
# Introduction of Variability

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- If there are small differences between scores, then the variability is small, and
- if there are large differences between scores, then the variability is large.

# Adult male heights and weights



# The two purposes of a good measure of variability

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  - It tells whether the scores are clustered close together or are spread out over a large distance.

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2. Variability measures how well an individual score (or group of scores) represents the entire distribution.
  - This aspect of variability is very important for inferential statistics, in which relatively small samples are used to answer questions about populations.
  - Variability provides information about how much error to expect if you are using a sample to represent a population.

# Common measures of variability

# Common measures of variability

- Variance and Standard Deviation

# Common measures of variability

- Variance and Standard Deviation
- Range

# Common measures of variability

- Variance and Standard Deviation
- Range
- Interquartile range

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# Defining Standard Deviation and Variance

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- The standard deviation is the most commonly used and the most important measure of variability.
- Standard deviation uses the mean of the distribution as a reference point and measures variability by considering the distance between each score and the mean.
- In simple terms, the standard deviation provides a measure of the standard, or *average distance* from the mean, and describes whether the scores are clustered closely around the mean or are widely scattered.

# The calculation of variance and standard deviation

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- Given the following  $N = 5$  scores

$$\begin{array}{r} \hline X \\ 1 \\ 9 \\ 5 \\ 8 \\ 7 \\ \hline \end{array}$$

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- Although the concept of standard deviation is straightforward, the actual equations tend to be more complex.
- Therefore, we begin by looking at the logic that leads to these equations.
- If you remember that our goal is to measure the standard, or typical, distance from the mean, then this logic and the equations that follow should be easier to remember.

# The calculation of variance and standard deviation

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- **Deviation** (离差) is distance from the mean:

$$\text{deviation score} = X - \mu$$

# The calculation of variance and standard deviation

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$$\text{deviation score} = X - \mu$$

- The deviation for the previous scores are

X	Mean	Deviation
1	6	-5
9	6	3
5	6	-1
8	6	2
7	6	1

# The calculation of variance and standard deviation

Find the deviation  
(distance from the mean for each score)

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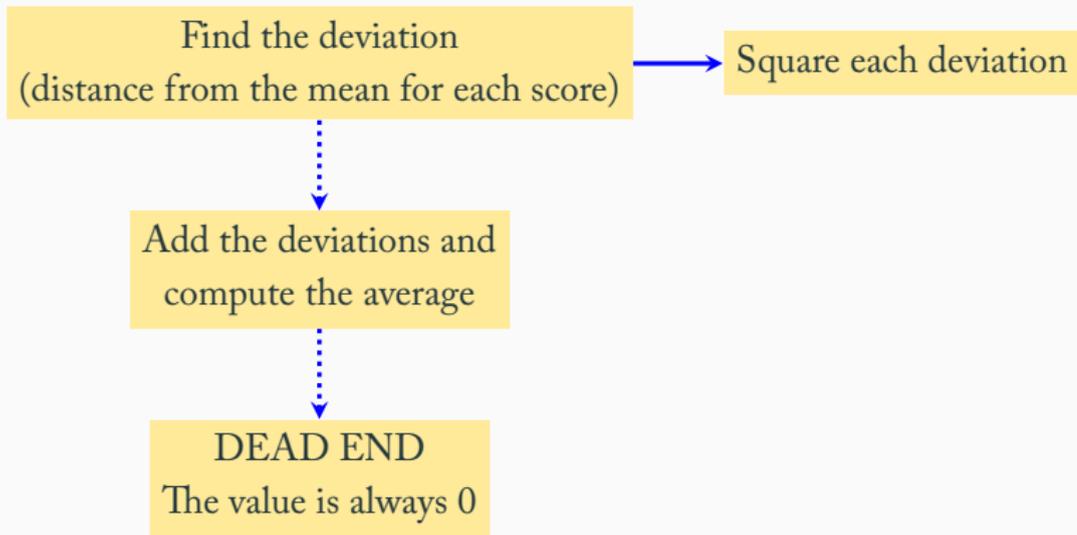
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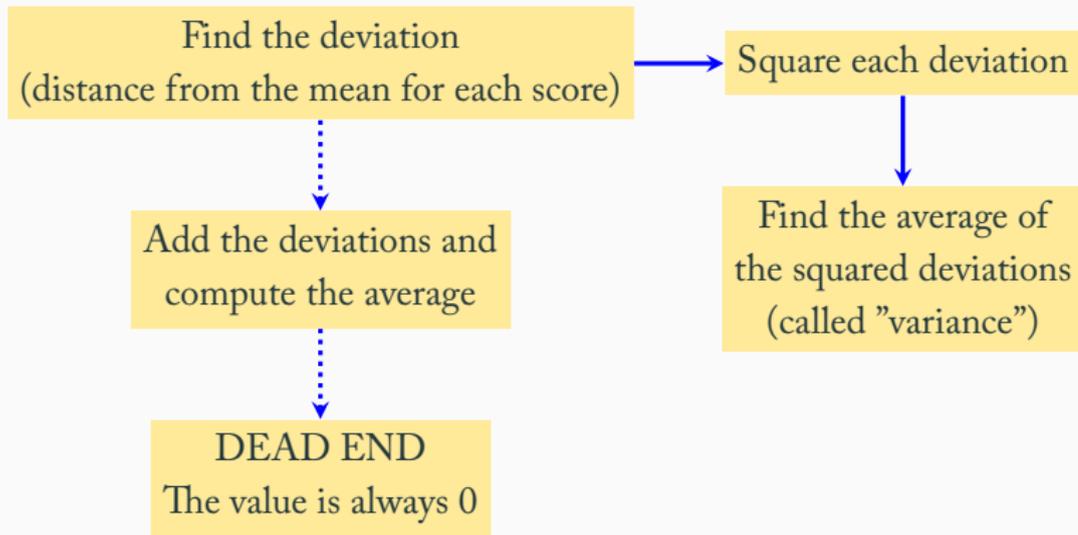
Add the deviations and  
compute the average

DEAD END  
The value is always 0

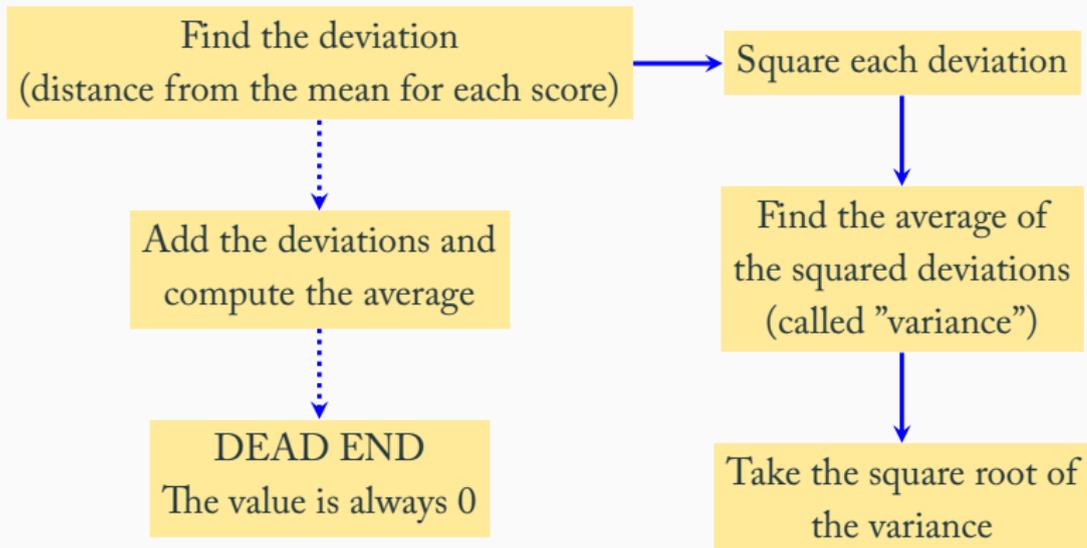
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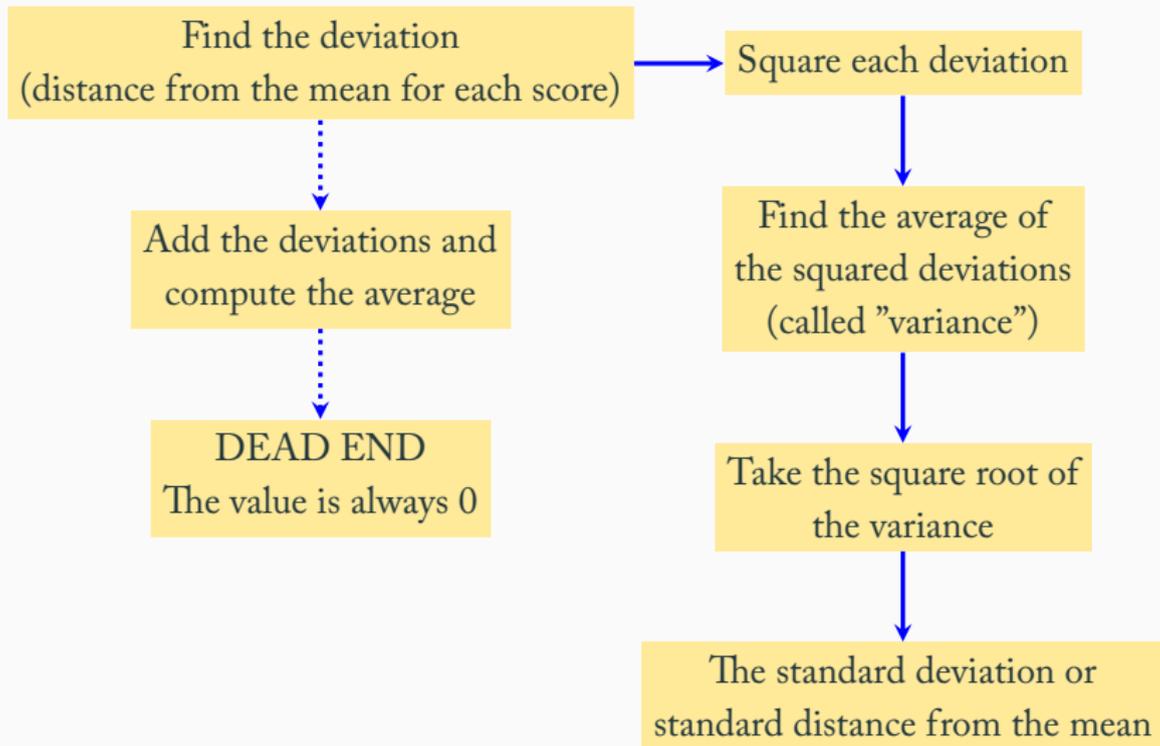
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- **Variance** (方差) equals the mean of the squared deviations. Variance is the average squared distance from the mean.
- **Standard deviation** (标准差) is the square root of the variance and provides a measure of the standard, or average distance from the mean.

$$\text{Standard deviation} = \sqrt{\text{variance}}$$

# The calculation of variance and standard deviation

## The calculation of variance and standard deviation

- Return to the previous  $N = 5$  scores

X	Mean	Deviation	Squared_Deviation
1	6	-5	25
9	6	3	9
5	6	-1	1
8	6	2	4
7	6	1	1

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- The sum of the squared deviation

$$\sum (X - \mu)^2 = 25 + 9 + 1 + 4 + 1 = 40$$

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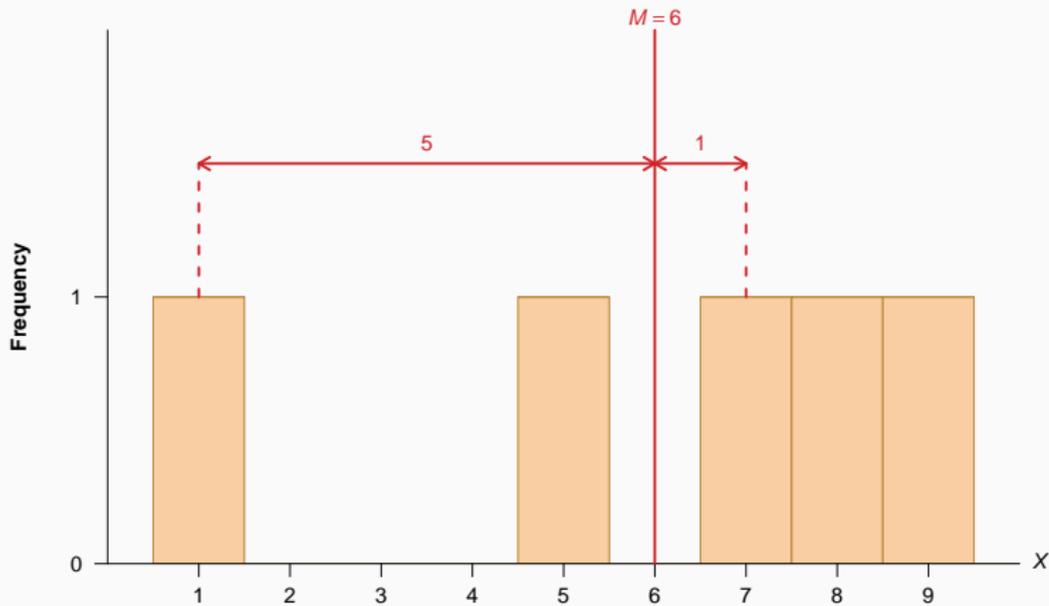
- The variance is

$$\begin{aligned}\text{Variance} &= \frac{\text{Sum of Squared Deviation}}{N} \\ &= \frac{40}{5} = 8\end{aligned}$$

- The standard deviation is

$$\begin{aligned}\text{Standard Deviation} &= \sqrt{\text{Variance}} \\ &= \sqrt{8} = 2.828\end{aligned}$$

# The calculation of variance and standard deviation



# The Sum of Squared Deviations (SS)

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$$\begin{aligned}\text{Variance} &= \text{mean squared deviation} \\ &= \frac{\text{sum of squared deviations}}{\text{number of scores}}\end{aligned}$$

- **SS**, or **sum of squares** (离差平方和), is the sum of the squared deviation scores.

# The Sum of Squared Deviations (SS)

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- Definitional formula

$$SS = \sum (X - \mu)^2 \quad (1)$$

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- Computational formula

$$SS = \sum X^2 - \frac{(\sum X)^2}{N} \quad (2)$$

## From Definitional formula to computational formula

$$\begin{aligned}SS &= \sum (X - \mu)^2 \\&= \sum (X^2 - 2 \cdot X \cdot \mu + \mu^2) \\&= \sum X^2 - 2 \cdot \mu \cdot \sum X + N \cdot \mu^2 \\&= \sum X^2 - 2 \cdot \frac{\sum X}{N} \cdot \sum X + N \cdot \frac{(\sum X)^2}{N^2} \\&= \sum X^2 - 2 \cdot \frac{(\sum X)^2}{N} + \frac{(\sum X)^2}{N} \\&= \sum X^2 - \frac{(\sum X)^2}{N}\end{aligned}$$

# Sum of squares: Definitional formula

## Sum of squares: Definitional formula

- Suppose a data set of  $N = 4$  scores

```
X <- c(1, 0, 6, 1)
Mean <- mean(X)
Deviation <- X - Mean
Squared_Deviation <- Deviation ^ 2
SS <- sum(Squared_Deviation)
SS
## [1] 22
```

## Sum of squares: Computational formula

## Sum of squares: Computational formula

- Return to the same data set

```
X <- c(1, 0, 6, 1)
Sum_Sqr_X <- sum(X ^ 2)
Sqr_Sum_X <- (sum(X)) ^ 2
N <- length(X)
SS <- Sum_Sqr_X - Sqr_Sum_X / N
SS
## [1] 22
```

# Final Formulas and Notation

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- **Population variance** (总体方差) is represented by the symbol  $\sigma^2$  and equals the mean squared distance from the mean.

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- Population variance is obtained by dividing the sum of squares by  $N$ .

$$\text{population variance} = \sigma^2 = \frac{SS}{N} \quad (3)$$

# Final Formulas and Notation

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- **Population standard deviation** (总体标准差) is represented by the symbol  $\sigma$  and equals the square root of the population variance.

$$\begin{aligned} & \text{population standard deviation} \\ & = \sigma = \sqrt{\sigma^2} \\ & = \sqrt{\frac{SS}{N}} \end{aligned} \tag{4}$$

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# Formulas for Sample Sum of Squares

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- The goal of inferential statistics is to use the limited information from samples to draw general conclusions about populations.

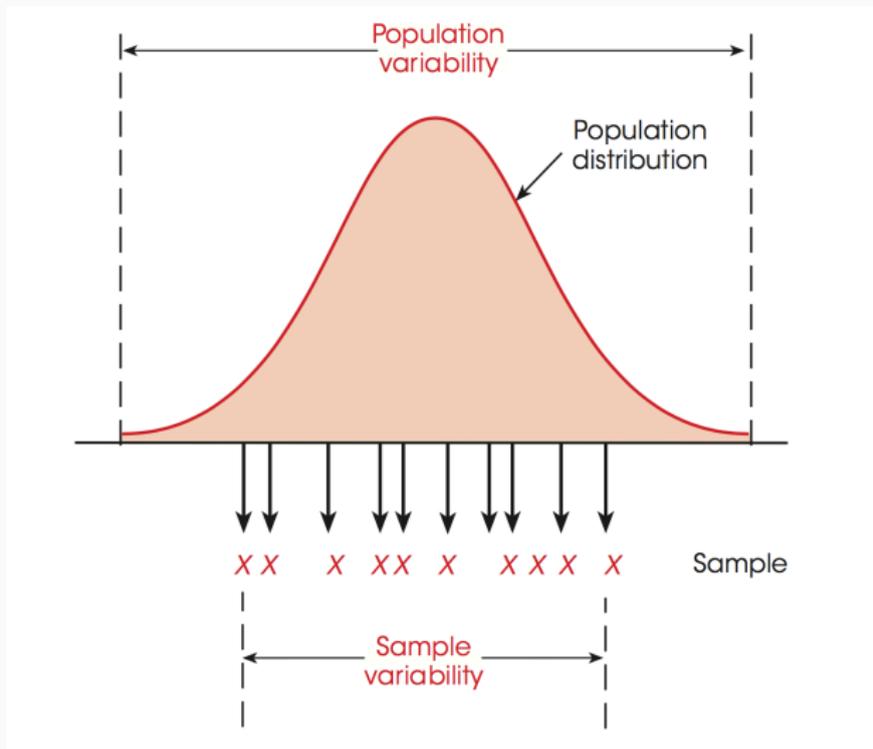
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- The goal of inferential statistics is to use the limited information from samples to draw general conclusions about populations.
- The basic assumption of this process is that samples should be representative of the populations from which they come.
- This assumption poses a special problem for variability because samples consistently tend to be less variable than their populations.

# The Problem with Sample Variability



# Formulas for Sample Sum of Squares

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- However, these extreme values are unlikely to be obtained when you are selecting a sample, which means that the sample variability is relatively small.
- The fact that a sample tends to be less variable than its population means that sample variability gives a **biased** estimate of population variability.
- Fortunately, the bias in sample variability is consistent and predictable, which means it can be corrected.

# Formulas for Sample Sum of Squares

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- The definitional formula for SS for a sample is

$$SS = \sum (X - M)^2 \quad (5)$$

- The computational formula for SS for a sample is

$$SS = \sum X^2 - \frac{(\sum X)^2}{n} \quad (6)$$

# Sample Variance and Standard Deviation

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- **Sample standard deviation** (样本标准差) is represented by the symbol  $s$  and equal the square root of the sample variance.

$$\begin{aligned} & \text{sample standard deviation} \\ =s & = \sqrt{s^2} = \sqrt{\frac{SS}{n - 1}} \end{aligned} \quad (8)$$

# Sample Variance and Standard Deviation

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## Sample Variance and Standard Deviation

- Remember that the formulas for sample variance and standard deviation were constructed so that the sample variability would provide a good estimate of population variability.
- For this reason, the sample variance is often called estimated population variance (整体方差的估计), and the sample standard deviation is called estimated population standard deviation (整体标准差的估计).
- When you have only a sample to work with, the variance and standard deviation for the sample provide the best possible estimates of the population variability.

# Sample Variability and Degrees of Freedom

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- Although the concept of a deviation score and the calculation SS are almost exactly the same for samples and populations, the minor differences in notation are really very important.
- Specifically, with a population, you find the deviation for each score by measuring its distance from the population mean.

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## Sample Variability and Degrees of Freedom

- With a sample, on the other hand, the value of  $\mu$  is unknown and you must measure distances from the sample mean.
- Because the value of the sample mean varies from one sample to another, you must first compute the sample mean before you can begin to compute deviations.
- However, calculating the value of  $M$  places a restriction on the variability of the scores in the sample.

## Sample Variability and Degrees of Freedom

- Suppose we select a sample of  $n = 3$  scores and compute a mean of  $M = 5$ .

---

X	A sample of $n = 3$ scores with a mean of $M = 5$
---	---

---

2

9

\_\_\_

← What is this third score?

---

# Sample Variability and Degrees of Freedom

## Sample Variability and Degrees of Freedom

- For a sample of  $n$  scores, the **degrees of freedom** (自由度), or **df**, for the sample variance are defined as  $df = n - 1$ .

## Sample Variability and Degrees of Freedom

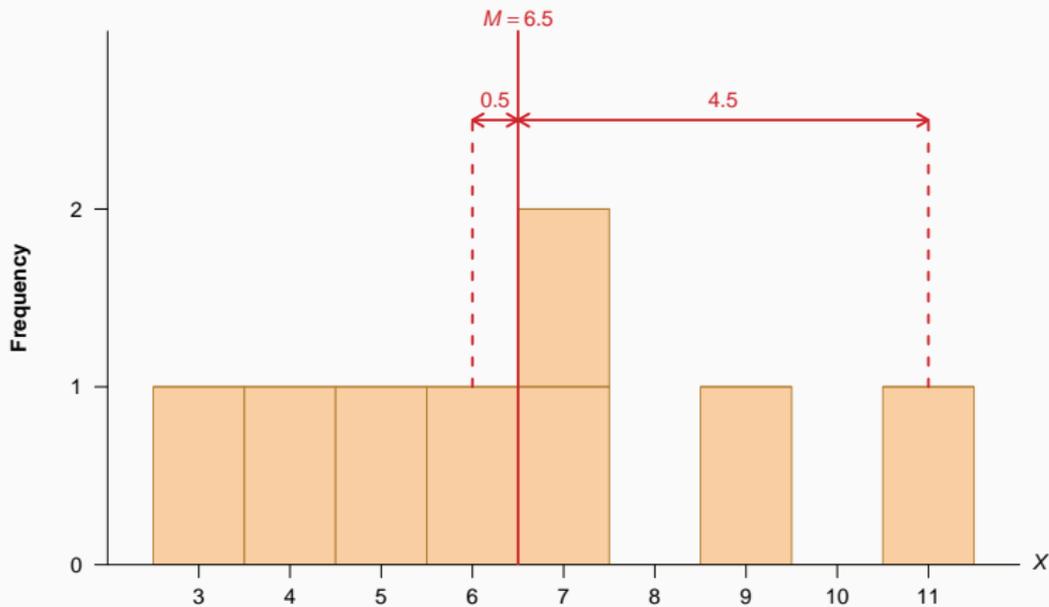
- For a sample of  $n$  scores, the **degrees of freedom** (自由度), or **df**, for the sample variance are defined as  $df = n - 1$ .
- The degrees of freedom determine the number of scores in the sample that are independent and free to vary.

## Sample Variability and Degrees of Freedom

- To calculate sample variance (mean squared deviation), we find the sum of the squared deviations (SS) and divide by the number of scores that are free to vary. This number is  $n - 1 = df$ .
- Thus, the formula for sample variance is

$$s^2 = \frac{\text{sum of squared deviation}}{\text{numbers of scores free to vary}} = \frac{SS}{df} = \frac{SS}{n - 1}$$

# Presenting Standard Deviation in Graphs



# The Sample Variance and Standard Deviation

```
X <- c(4, 6, 5, 11, 7, 9, 7, 3)
```

# The Sample Variance and Standard Deviation

```
X <- c(4, 6, 5, 11, 7, 9, 7, 3)
```

```
SS <- sum((X - mean(X))^2)
```

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SS <- sum(X^2) - (sum(X))^2 / length(X)
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```
s_sqr <- SS / (length(X) - 1)
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```
s <- sqrt(s_sqr)
```

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```
## [1] 6.857143
```

```
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```

# The Sample Variance and Standard Deviation

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- Use the `var()` function in R

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```

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sd(X)
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# Biased and Unbiased Statistics

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## Biased and Unbiased Statistics

- A sample statistic **unbiased** (无偏) if the average value of the statistic is equal to the population parameter.
- The average value of the statistic is obtained from all the possible samples for a specific sample size,  $n$ .
- A sample statistic is **biased** (有偏的) if the average value of the statistic either underestimates or overestimates the corresponding population parameter.

# Biased and Unbiased Statistics

## Biased and Unbiased Statistics

- Suppose a population that consists of exactly  $N = 3$  scores

```
X <- c(0, 3, 9)
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- This population has a mean of  $\mu = 4$  and,
- This population has a variance of  $\sigma^2 = 14$ .

## Biased and Unbiased Statistics

- Suppose a population that consists of exactly  $N = 3$  scores

```
X <- c(0, 3, 9)
```

- This population has a mean of  $\mu = 4$  and,
- This population has a variance of  $\sigma^2 = 14$ .
- Next, we select samples of  $n = 2$  scores from this population.

# Biased and Unbiased Statistics

## Biased and Unbiased Statistics

- In fact, we obtain every single possible sample with  $n = 2$ .

	Score_1	Score_2	Mean
1	0	0	0.00
2	3	0	1.50
3	9	0	4.50
4	0	3	1.50
5	3	3	3.00
6	9	3	6.00
7	0	9	4.50
8	3	9	6.00
9	9	9	9.00

# Biased and Unbiased Statistics

## Biased and Unbiased Statistics

- The biased and unbiased variances for each sample are:

	Score_1	Score_2	Mean	Biased	Unbiased
1	0	0	0.00	0.00	0.00
2	3	0	1.50	2.25	4.50
3	9	0	4.50	20.25	40.50
4	0	3	1.50	2.25	4.50
5	3	3	3.00	0.00	0.00
6	9	3	6.00	9.00	18.00
7	0	9	4.50	20.25	40.50
8	3	9	6.00	9.00	18.00
9	9	9	9.00	0.00	0.00

# Biased and Unbiased Statistics

# Biased and Unbiased Statistics

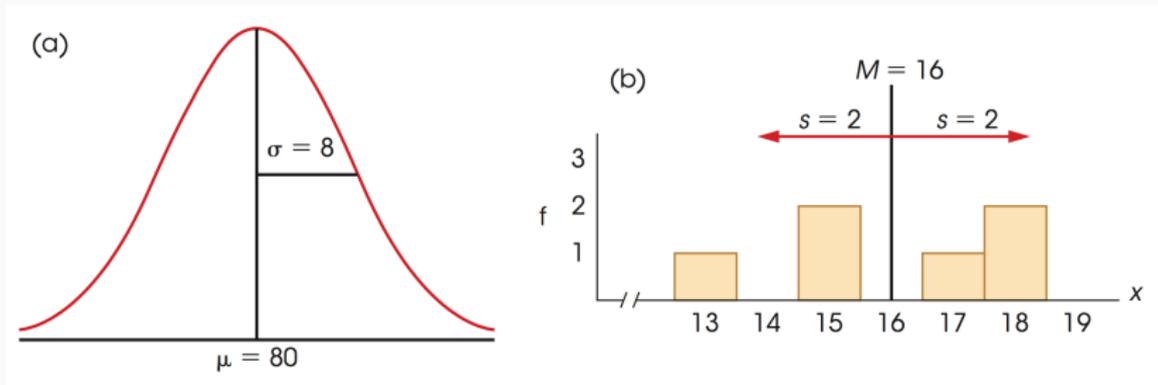
- The mean of sample mean and of the two variances are

```
## Mean
##      4
## Biased
##       7
## Unbiased
##      14
```

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# Presenting the Mean and Standard Deviation



# Transformations of Scale

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# Transformations of Scale

- Adding a constant to each score does not change the standard deviation.
- Multiplying each score by a constant causes the standard deviation to be multiplied by the same constant.

# Reporting the Standard Deviation

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- In reporting the results of a study, the researcher often provides descriptive information for both central tendency and variability.
- The dependent variables in psychology research are often numerical values obtained from measurements on interval or ratio scales.
- With numerical scores the most common descriptive statistics are the mean (central tendency) and the standard deviation (variability), which are usually reported together.

# Reporting the Standard Deviation

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- In many journals, especially those following APA style, the symbol SD is used for the sample standard deviation.

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- In many journals, especially those following APA style, the symbol *SD* is used for the sample standard deviation.

Children who viewed the violent cartoon displayed more aggressive responses ( $M = 12.45$ ,  $SD = 3.7$ ) than those who viewed the control cartoon ( $M = 4.22$ ,  $SD = 1.04$ ).

# Reporting the Standard Deviation

## Reporting the Standard Deviation

- When reporting the descriptive measures for several groups, the findings may be summarized in a table.

	Type of Video Game	
	Violent	Control
Males	M=7.72 SD=2.43	M=4.34 SD=2.16
Females	M=2.47 SD=0.92	M=1.61 SD=0.68

## Reporting the Standard Deviation

- When reporting the descriptive measures for several groups, the findings may be summarized in a table.

	Type of Video Game	
	Violent	Control
Males	M=7.72 SD=2.43	M=4.34 SD=2.16
Females	M=2.47 SD=0.92	M=1.61 SD=0.68

- Sometimes the table also indicates the sample size,  $n$ , for each group.

# Standard Deviation and Descriptive Statistics

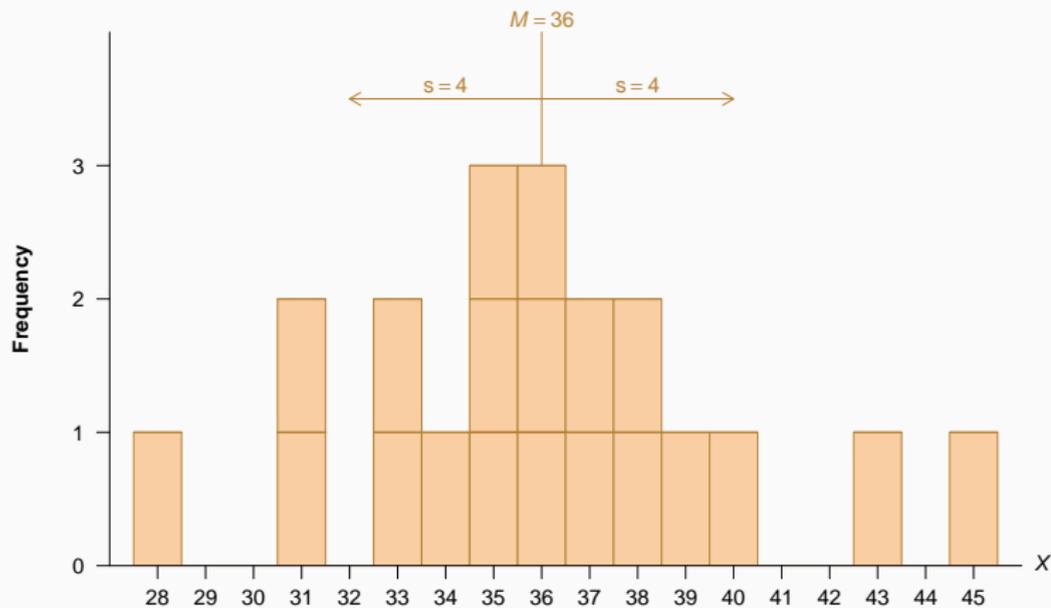
# Standard Deviation and Descriptive Statistics

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# Standard Deviation and Descriptive Statistics

- **Describing an Entire Distribution.** Because the mean identifies the center of a distribution and the standard deviation describes the average distance from the mean, these two values should allow you to create a reasonably accurate image of the entire distribution.
- **Describing the Location of Individual Scores.** Knowing the mean and standard deviation should also allow you to describe the relative location of any individual score within the distribution.

# Standard Deviation and Descriptive Statistics



# Variance and Inferential Statistics

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- Variability plays an important role in the inferential process because the variability in the data influences how easy it is to see patterns.
- In general, low variability means that existing patterns can be seen clearly, whereas high variability tends to obscure any patterns that might exist.

# Variance and Inferential Statistics

# Variance and Inferential Statistics

- Experiment A

Treatment_1	Treatment_2
35	39
34	40
36	41
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- Experiment B

Treatment_1	Treatment_2
31	46
15	21
57	61
37	32

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35	39
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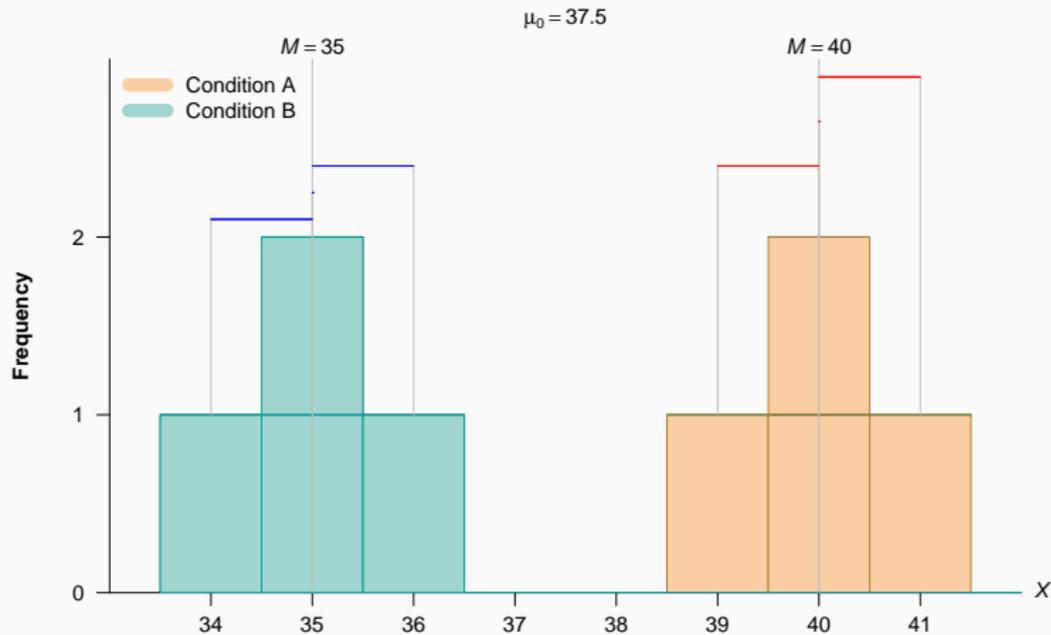
- Mean (Treatment\_1) = 35
- Mean (Treatment\_2) = 40

- Experiment B

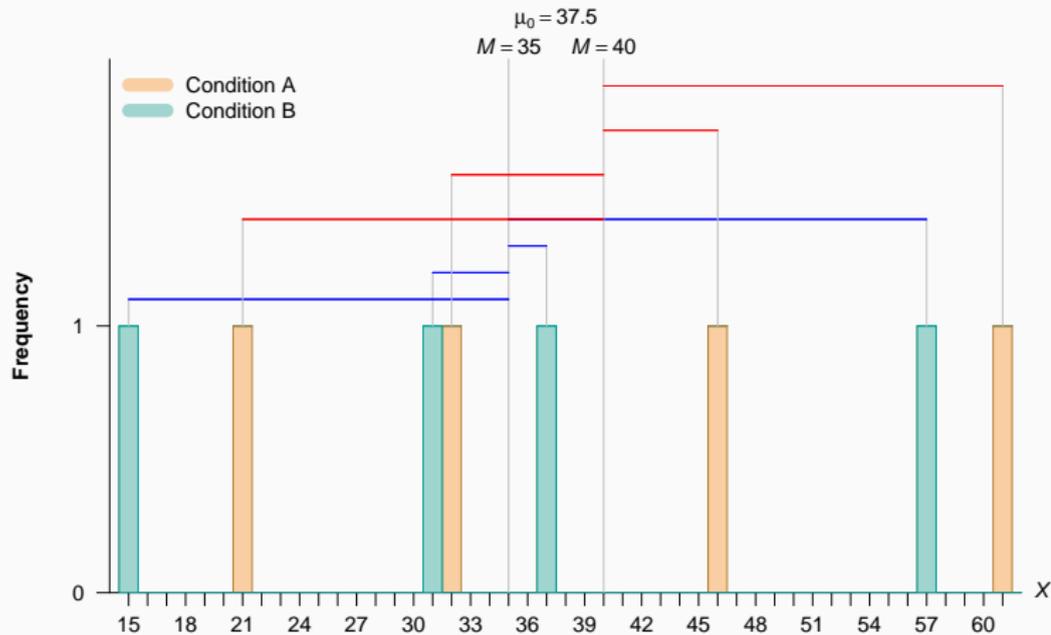
Treatment_1	Treatment_2
31	46
15	21
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37	32

- Mean (Treatment\_1) = 35
- Mean (Treatment\_2) = 40

# Variance and Inferential Statistics



# Variance and Inferential Statistics



# Table of Contents

1. Introduction of Variability
2. Variance and Standard Deviation for a Population
3. Variance and Standard Deviation for a Sample
4. More about Variance and Standard Deviation
5. Range and interquartile range

# The Range

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$$\text{Range} = \text{URL for } X_{max} - \text{LRL for } X_{min}$$

- When the scores are whole numbers, the range can also be defined as the number of measurement categories.

$$\text{Range} = X_{max} - X_{min} + 1$$

# The Range function in R

```
(X <- seq(2, 17, by = 3))
```

```
range(X)
```

```
min(X)
```

```
max(X)
```

```
## [1] 2 5 8 11 14 17
```

```
## [1] 2 17
```

```
## [1] 2
```

```
## [1] 17
```

# Interquartile range

## Interquartile range

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- The **interquartile range (IQR)** (四分位距) is defined as  $IQR = Q3 - Q1$ .

# Interquartile range

X

```
## [1]  2  5  8 11 14 17
```

# Interquartile range

```
X
```

```
## [1]  2  5  8 11 14 17
```

```
quantile(X)
```

```
##      0%    25%    50%    75%   100%
```

```
##  2.00  5.75  9.50 13.25 17.00
```

# Interquartile range

```
X
```

```
## [1]  2  5  8 11 14 17
```

```
quantile(X)
```

```
##      0%   25%   50%   75%  100%  
##  2.00  5.75  9.50 13.25 17.00
```

```
IQR(X)
```

```
## [1] 7.5
```

Questions?