Chapter 5. z-Scores

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Introduction of z-Scores

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- In this chapter, we introduce a statistical technique that uses the mean and the standard deviation to transform each score (X value) into a z-score or a **standard score** (标准分).

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- Now we shift attention to the individual scores within a distribution.
- In this chapter, we introduce a statistical technique that uses the mean and the standard deviation to transform each score (X value) into a z-score or a **standard score** (标准分).
- The purpose of z-scores, or standard scores, is to identify and describe the exact location of each score in a distribution.

Introduction of z-Scores: An example

Suppose

You received a score of $X = 76$ on a statistics exam.

Introduction of z-Scores: An example

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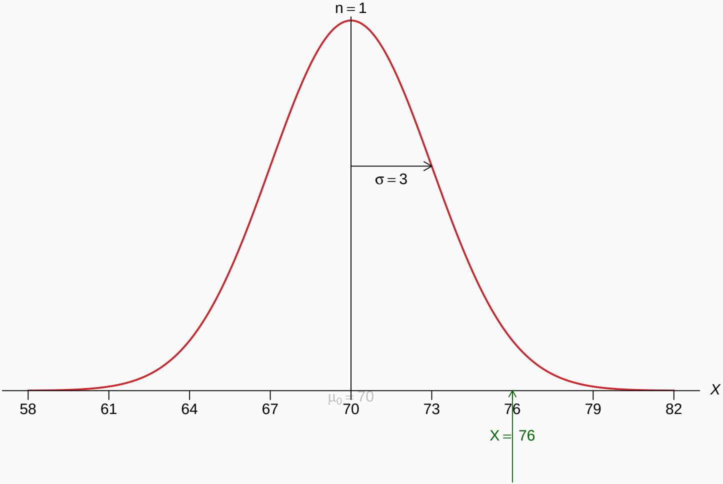
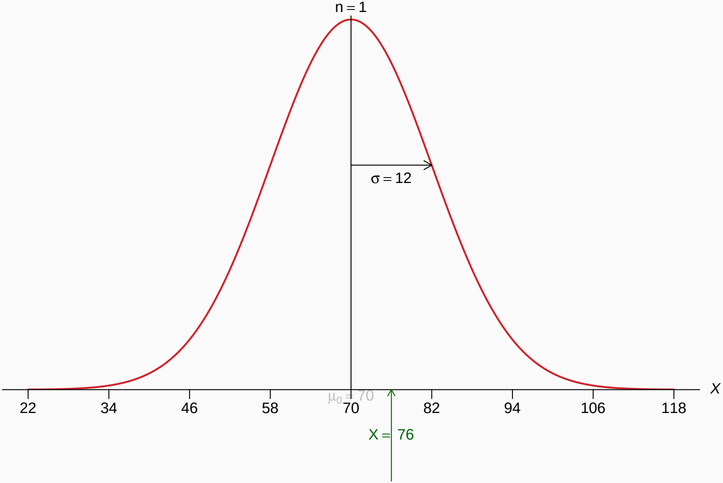
You received a score of $X = 76$ on a statistics exam.

How did you do?

Introduction of z-Scores



Introduction of z-Scores



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- These original, unchanged scores that are the direct result of measurement are called **raw scores** (原始分数).
- To make raw scores more meaningful, they are often transformed into new values that contain more information. This transformation is one purpose for z-scores.
- In particular, we transform X values into z-scores so that the resulting z-scores tell exactly where the original scores are located.

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- A second purpose for z-scores is to standardize an entire distribution.
- A common example of a standardized distribution is the distribution of IQ scores.
- Although there are several different tests for measuring IQ, the tests usually are standardized so that they have a mean of 100 and a standard deviation of 15.

The two useful purposes of z-scores

1. Each z-score tells the exact location of the original *X* value within the distribution.

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1. Each z-score tells the exact location of the original X value within the distribution.
2. The z-scores form a standardized distribution that can be directly compared to other distributions that also have been transformed into z-scores.

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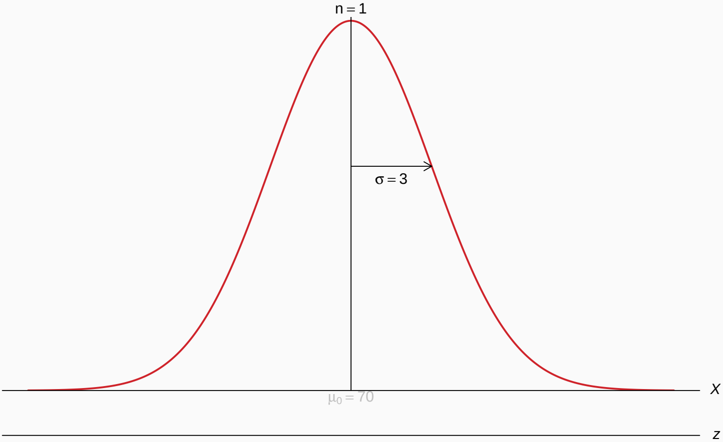
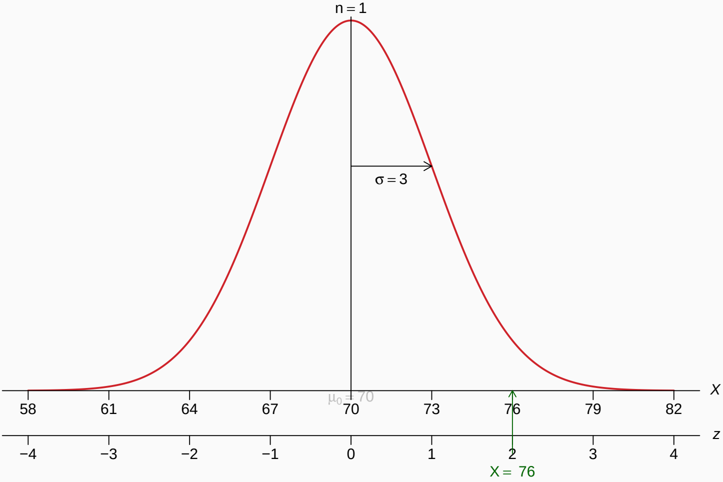
- A **z-score** (z 分数) specifies the precise location of each X value within a distribution.
- The sign of the z-score (+ or) signifies whether the score is above the mean (positive) or below the mean (negative).
- The numerical value of the z-score specifies the distance from the mean by counting the number of standard deviations between X and μ.
- The locations identified by z-scores are the same for all distributions, no matter what mean or standard deviation the distributions may have.

z-Scores and Location in a Distribution



Introduction of z-Scores



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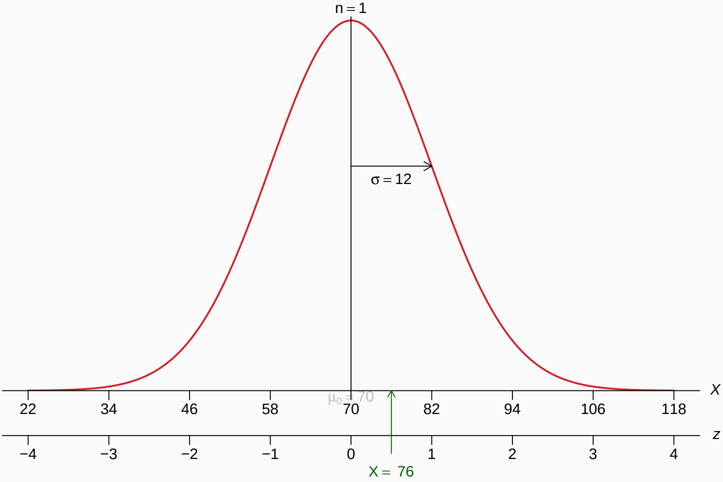

The z-Score Formula

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- The formula for transforming scores into z-scores is

$$z = \frac{X - \mu}{\sigma} \quad (1)$$

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The z-Score Formula

- The formula for transforming scores into z-scores is

$$z = \frac{X - \mu}{\sigma} \tag{1}$$

- The numerator of the equation, $X - \mu$, is a deviation score;
- it measures the distance in points between X and μ and indicates whether X is located above or below the mean.
- The deviation score is then divided by σ because we want the z-score to measure distance in terms of **standard deviation units** (标准化的离差单位).

The Relationships Between z, X, μ, and σ

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- What z-score corresponds to a score of $X = 130$ in this distribution?

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```r
mu <- 100
sigma <- 10
X <- 130
z <- (X - mu) / sigma
```

The Relationships Between z, X, μ, and σ

- A distribution of scores has a mean of $\mu = 100$ and a standard deviation of $\sigma = 10$.
- What z-score corresponds to a score of $X = 130$ in this distribution?

```
mu <- 100
sigma <- 10
X <- 130
z <- (X - mu) / sigma
```

```
## [1] 3
```

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- A distribution of scores has a mean of $\mu = 86$ and a standard deviation of $\sigma = 7$.

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- A distribution of scores has a mean of $\mu = 86$ and a standard deviation of $\sigma = 7$.
- What z-score corresponds to a score of $X = 95$ in this distribution?

```
mu <- 86
sigma <- 7
X <- 95
z <- (X - mu) / sigma
```

The Relationships Between z, X, μ, and σ

- A distribution of scores has a mean of $\mu = 86$ and a standard deviation of $\sigma = 7$.
- What z-score corresponds to a score of $X = 95$ in this distribution?

```
mu <- 86
sigma <- 7
X <- 95
z <- (X - mu) / sigma
```

```
## [1] 1.285714
```

The z-Score Formula

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- A **z-score** establishes a relationship between the **raw score** (X), **mean** (μ), and **standard deviation** (σ).

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- A **z-score** establishes a relationship between the **raw score** (X), **mean** (μ), and **standard deviation** (σ).
- Knowing any three of the four numbers, we can solve the fourth one.

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- For a distribution with a mean of $\mu = 60$ and $\sigma = 8$,
- What X value corresponds to a z-score of $z = 1.50$?

```
mu <- 60
sigma <- 8
z <- -1.50
X <- z * sigma + mu
```

Determining a Raw Score (X) from a z-Score

- For a distribution with a mean of $\mu = 60$ and $\sigma = 8$,
- What X value corresponds to a z-score of $z = 1.50$?

```
mu <- 60
sigma <- 8
z <- -1.50
X <- z * sigma + mu
```

```
## [1] 48
```

Determining σ from a z-Score

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- In a population with a mean of $\mu = 65$, a score of $X = 59$ corresponds to $z = -2.00$,

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- What is the standard deviation for the population?

```
mu <- 65
X <- 59
z <- -2.00
sigma <- (X - mu) / z
```

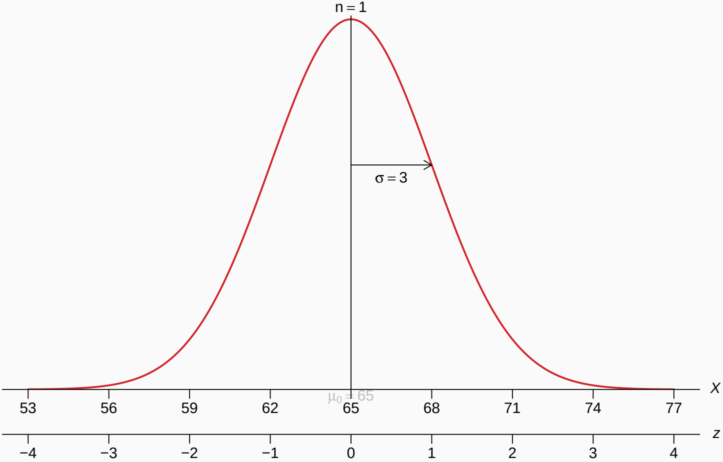
Determining σ from a z-Score

- In a population with a mean of $\mu = 65$, a score of $X = 59$ corresponds to $z = -2.00$,
- What is the standard deviation for the population?

```
mu <- 65
X <- 59
z <- -2.00
sigma <- (X - mu) / z
```

```
## [1] 3
```

Determining σ from a z-Score



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- What is the mean for the population?

```
sigma <- 6
X <- 33
z = + 1.50
mu <- X - z * sigma
```

Determining μ from a z-Score

- In a population with a standard deviation of $\sigma = 6$, a score of $X = 33$ corresponds to $z = +1.50$,
- What is the mean for the population?

```
sigma <- 6
X <- 33
z = + 1.50
mu <- X - z * sigma
```

```
## [1] 24
```

Determining μ and σ from two z-Scores

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- In a population distribution, a score of $X = 64$ has a z-score of $z = 0.50$ and a score of $X = 72$ has a z-score of $z = 1.50$,

Determining μ and σ from two z-Scores

- In a population distribution, a score of $X = 64$ has a z-score of $z = 0.50$ and a score of $X = 72$ has a z-score of $z = 1.50$,
- What are the values for the population mean and standard deviation?

Determining μ and σ from two z-Scores

- In a population distribution, a score of $X = 64$ has a z-score of $z = 0.50$ and a score of $X = 72$ has a z-score of $z = 1.50$,
- What are the values for the population mean and standard deviation?
- The formula

$$z = \frac{X - \mu}{\sigma}$$

$$\Downarrow$$

$$z \cdot \sigma + \mu = X$$

Determining μ and σ from two z-Scores

```
z <- c(0.5, 1.5)
m <- c(  1,   1)
X <- c( 64,  72)
```

Determining μ and σ from two z-Scores

```r
z <- c(0.5, 1.5)
m <- c(  1,   1)
X <- c( 64,  72)
```

```r
cbind(z, m, X)
```

```
##        z m  X
## [1,] 0.5 1 64
## [2,] 1.5 1 72
```

Determining μ and σ from two z-Scores

```r
z <- c(0.5, 1.5)
m <- c(  1,   1)
X <- c( 64,  72)
```

```r
cbind(z, m, X)
```

```
##        z m  X
## [1,] 0.5 1 64
## [2,] 1.5 1 72
```

```r
(coef <- solve(cbind(z, m), X))
```

```
##  z  m
##  8 60
```

Determining μ and σ from two z-Scores

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- In a population distribution, a score of $X = 54$ corresponds to $z = +2.00$ and a score of $X = 42$ corresponds to $z = -1.00$.

Determining μ and σ from two z-Scores

- In a population distribution, a score of $X = 54$ corresponds to $z = +2.00$ and a score of $X = 42$ corresponds to $z = -1.00$.
- What are the values for the mean and the standard deviation for the population?

Determining μ and σ from two z-Scores

```
z <- c( 2, -1)
m <- c( 1,  1)
X <- c(54, 42)
```

Determining μ and σ from two z-Scores

```r
z <- c( 2, -1)
m <- c( 1,  1)
X <- c(54, 42)
```

```r
cbind(z, m, X)
```

```
##       z m  X
## [1,]  2 1 54
## [2,] -1 1 42
```

Determining μ and σ from two z-Scores

```
z <- c( 2, -1)
m <- c( 1,  1)
X <- c(54, 42)
```

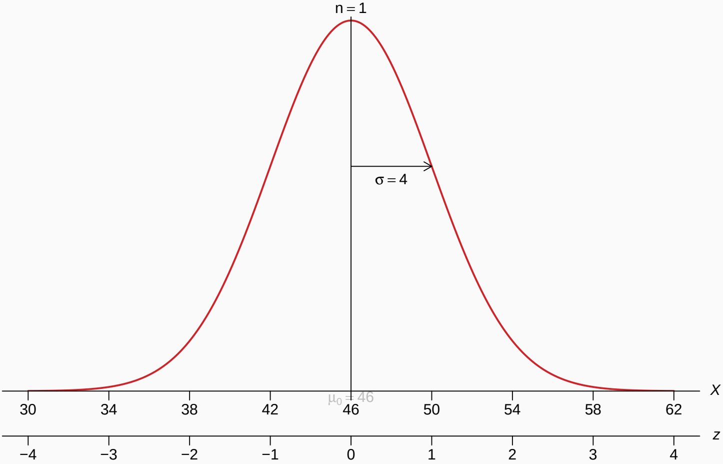
```
cbind(z, m, X)
```

```
##       z m  X
## [1,]  2 1 54
## [2,] -1 1 42
```

```
(coef <- solve(cbind(z, m), X))
```

```
##  z  m
##  4 46
```

Determining μ and σ from two z-Scores



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Properties of the distribution of z-scores

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- It is possible to transform every X value in a distribution into a corresponding z-score.
- The result of this process is that the entire distribution of X values is transformed into a distribution of z-scores
- The new distribution of z-scores has characteristics that make the z-score transformation a very useful tool.
- Specifically, if every X value is transformed into a z-score, then the distribution of z-scores will have the following properties:

Properties of the distribution of z-scores

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- **Shape** (形状). The distribution of z-scores will have exactly the same shape as the original distribution of scores.

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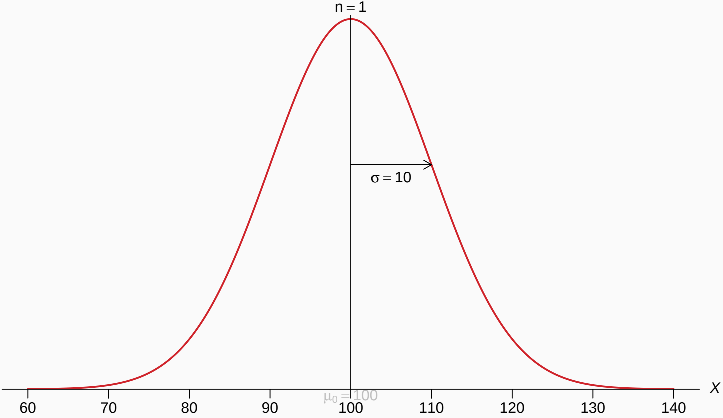
- **The Mean** (平均值). The z-score distribution will always have a mean of zero.

Properties of the distribution of z-scores

- **Shape** (形状). The distribution of z-scores will have exactly the same shape as the original distribution of scores.
- **The Mean** (平均值). The z-score distribution will always have a mean of zero.
- **The Standard Deviation** (标准差). The distribution of z-scores will always have a standard deviation of 1.

Properties of the distribution of z-scores



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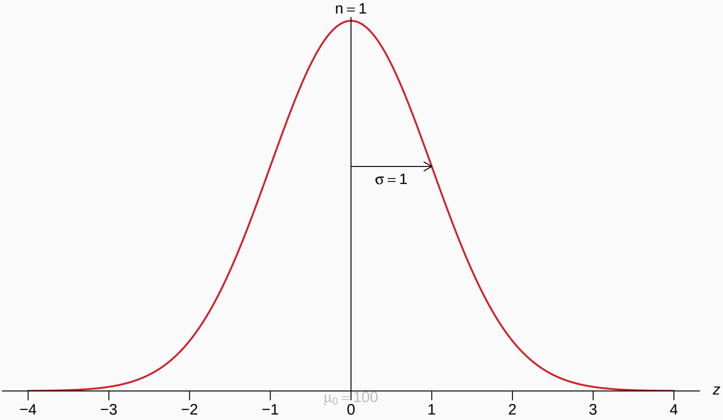

Using z-Scores to Standardize a Distribution

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- When any distribution (with any mean or standard deviation) is transformed into z-scores, the resulting distribution will always have a mean of $\mu = 0$ and a standard deviation of $\sigma = 1$.

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- When any distribution (with any mean or standard deviation) is transformed into z-scores, the resulting distribution will always have a mean of $\mu = 0$ and a standard deviation of $\sigma = 1$.
- Because all z-score distributions have the same mean and the same standard deviation, the z-score distribution is called a **standardized distribution** (标准化分布).

Using z-Scores to Standardize a Distribution

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- A standardized distribution is composed of scores that have been transformed to create predetermined values for μ and σ.

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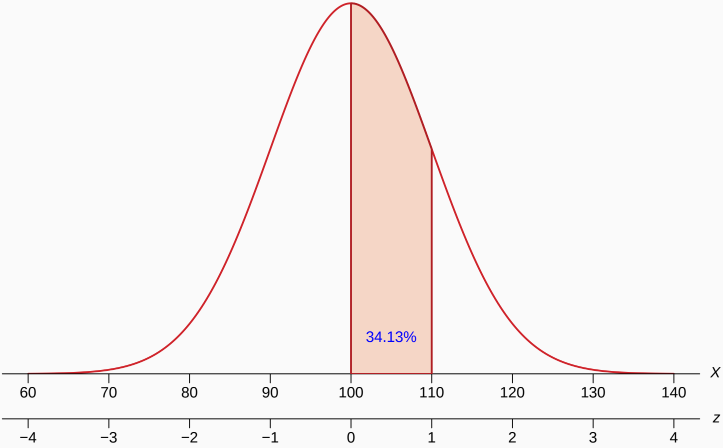
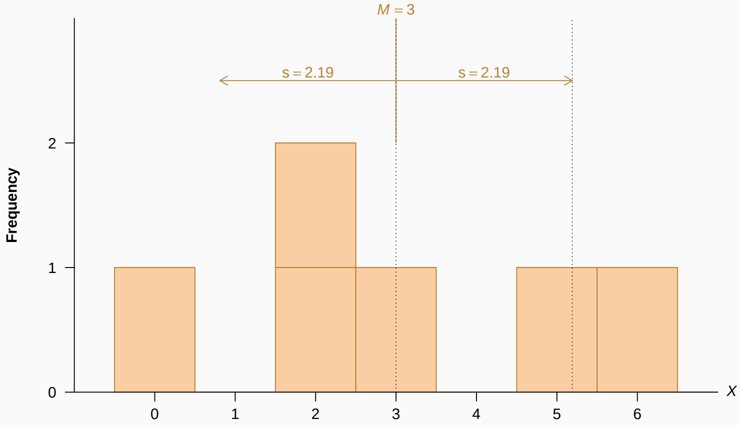
- A standardized distribution is composed of scores that have been transformed to create predetermined values for μ and σ.
- Standardized distributions are used to make dissimilar distributions comparable.
- A z-score distribution is an example of a standardized distribution with $\mu = 0$ and $\sigma = 1$.

z-Scores: A single distribution with two sets of labels



Demonstration of a z-Score Transformation



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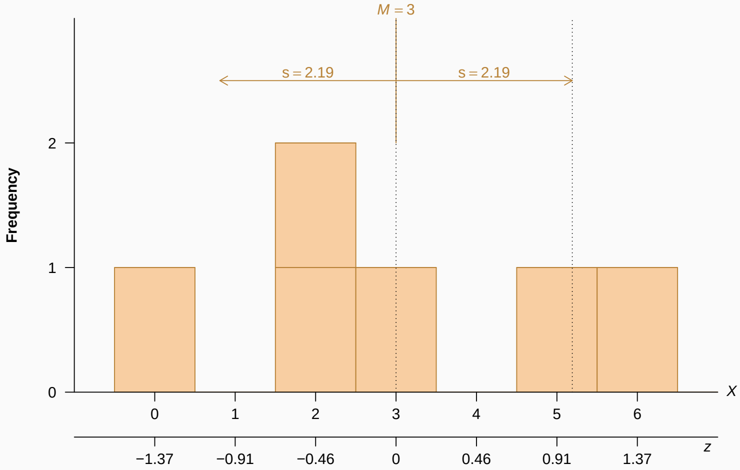

Using z-Scores for Making Comparisons

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- One advantage of standardizing distributions is that it makes it possible to compare different scores or different individuals even though they come from completely different distributions.

Using z-Scores for Making Comparisons

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- Suppose, for example, Dave received a score of $X = 60$ on a psychology exam and a score of $X = 56$ on a biology test.

Using z-Scores for Making Comparisons

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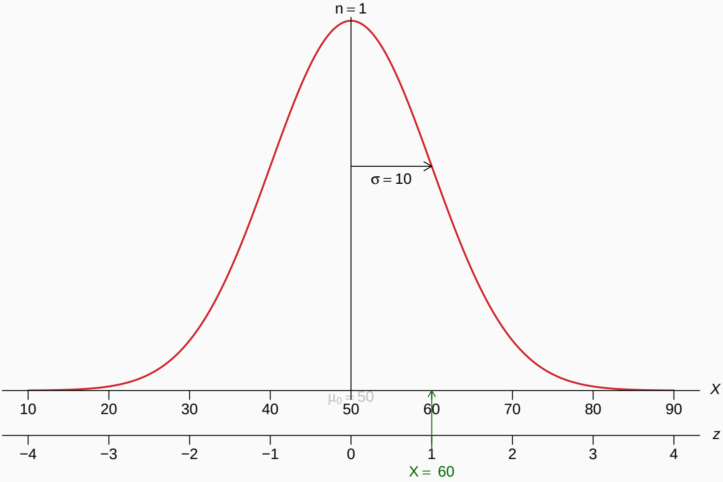
- For which course should Dave expect the better grade?.

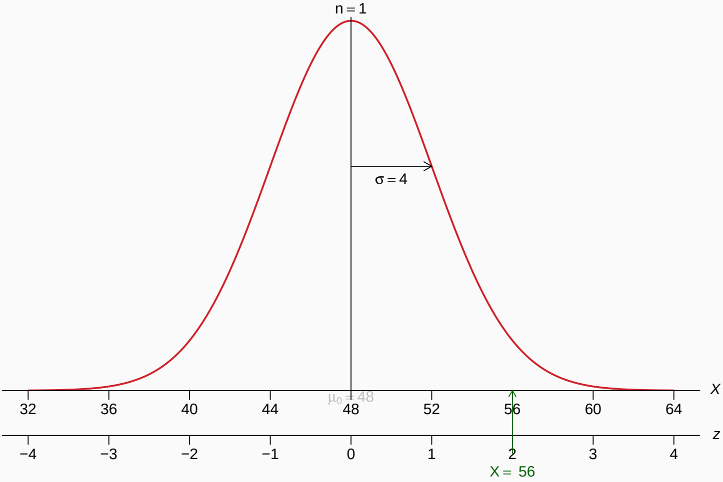
Using z-Scores for Making Comparisons

- Suppose, for example, Dave received a score of $X = 60$ on a psychology exam and a score of $X = 56$ on a biology test.
- For which course should Dave expect the better grade?.
- Suppose the psychology scores had $\mu = 50$ and $\sigma = 10$, and the biology scores had $\mu = 48$ and $\sigma = 4$.

Using z-Scores for Making Comparisons: Psychology



Using z-Scores for Making Comparisons: Biology



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To a Distribution with a Predetermined μ and σ

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To a Distribution with a Predetermined μ and σ

- Although z-score distributions have distinct advantages, many people find them cumbersome because they contain negative values and decimals.
- For this reason, it is common to standardize a distribution by transforming the scores into a new distribution with a predetermined mean and standard deviation that are whole round numbers.
- The goal is to create a new (standardized) distribution that has “simple” values for the mean and standard deviation but does not change any individual’s location within the distribution.

To a Distribution with a Predetermined μ and σ

- Although z-score distributions have distinct advantages, many people find them cumbersome because they contain negative values and decimals.
- For this reason, it is common to standardize a distribution by transforming the scores into a new distribution with a predetermined mean and standard deviation that are whole round numbers.
- The goal is to create a new (standardized) distribution that has “simple” values for the mean and standard deviation but does not change any individual’s location within the distribution.
- Standardized scores of this type are frequently used in psychological or educational testing.

To a Distribution with a Predetermined μ and σ

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- The original raw scores are transformed into z-scores.

To a Distribution with a Predetermined μ and σ

- The original raw scores are transformed into z-scores.
- The z-scores are then transformed into new X values so that the specific μ and σ are attained.

To a Distribution with a Predetermined μ and σ

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- For an psychological exam, the distribution of raw scores has a mean of $\mu = 57$ with $\sigma = 14$.

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- For an psychological exam, the distribution of raw scores has a mean of $\mu = 57$ with $\sigma = 14$.
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To a Distribution with a Predetermined μ and σ

- For an psychological exam, the distribution of raw scores has a mean of $\mu = 57$ with $\sigma = 14$.
- We would like to simplify the distribution by transforming all scores into a new, standardized distribution with $\mu = 50$ and $\sigma = 10$.
- Suppose two specific students: Maria, who has a raw score of $X = 64$ in the original distribution; and Joe, whose original raw score is $X = 43$.

To a Distribution with a Predetermined μ and σ

- For an psychological exam, the distribution of raw scores has a mean of $\mu = 57$ with $\sigma = 14$.
- We would like to simplify the distribution by transforming all scores into a new, standardized distribution with $\mu = 50$ and $\sigma = 10$.
- Suppose two specific students: Maria, who has a raw score of $X = 64$ in the original distribution; and Joe, whose original raw score is $X = 43$.
- What are their scores in the new distribution?

To a Distribution with a Predetermined μ and σ

	Original Scores $\mu = 57$ and $\sigma = 14$		z-Score Location		Standardized Scores $\mu = 50$ and $\sigma = 10$
Maria	$X = 64$	$\rightarrow$	$z = +0.50$	$\rightarrow$	$X = 55$
Joe	$X = 43$	$\rightarrow$	$z = -1.00$	$\rightarrow$	$X = 40$

To a Distribution with a Predetermined μ and σ

```
converter <- function(
  oldscore, oldmean, oldsd, newmean, newsd ){
     zscore <-  (oldscore - oldmean) / oldsd
     newscore <- zscore * newsd + newmean
     return(newscore)
}
```

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```
converter <- function(
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     zscore <-  (oldscore - oldmean) / oldsd
     newscore <- zscore * newsd + newmean
     return(newscore)
}
```

```
converter(64, 57, 14, 50, 10)
converter(43, 57, 14, 50, 10)

## [1] 55
## [1] 40
```

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Computing z-Scores for Samples

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- Similarly, each z-score can be transformed back into an X value, as follows:

$$X = M + z \times s \tag{3}$$

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- What is the X value corresponding to $z = +2.00$?

```
(z <- (35 - 40) / 10)
(X <- 2 * 10 + 40)
## [1] -0.5
## [1] 60
```

Standardizing a Sample Distribution

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  2. The sample of z-scores will have a mean of $M_z = 0$.
  3. The sample of z-scores will have a standard deviation of $s_z = 1$.

Standardizing a Sample Distribution

```r
X <- c(0, 2, 4, 4, 5)
```

Standardizing a Sample Distribution

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```
SS <- sum(X ^ 2) - (sum(X)) ^ 2 / length(X)
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s <- sqrt(SS / (length(X) - 1))
M <- mean(X)
```

```r
(z <- (X - M) / s)

## [1] -1.5 -0.5  0.5  0.5  1.0
```

Standardizing a Sample Distribution

```
scale(X)

##      [,1]
## [1,] -1.5
## [2,] -0.5
## [3,]  0.5
## [4,]  0.5
## [5,]  1.0
## attr(,"scaled:center")
## [1] 3
## attr(,"scaled:scale")
## [1] 2
```

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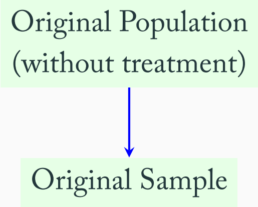
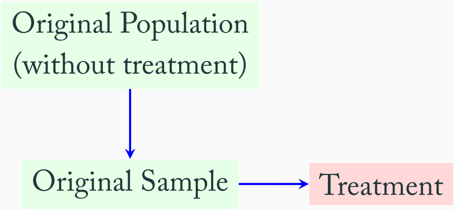
The process of a research study

Original Population
(without treatment)

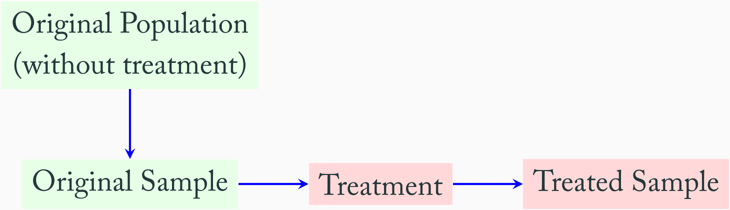
The process of a research study



The process of a research study



The process of a research study



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- A typical research study begins with a question about how a treatment will affect the individuals in a population.
- Because it is usually impossible to study an entire population, the researcher selects a sample and administers the treatment to the individuals in the sample.

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- If the individuals in the sample are noticeably different from the individuals in the original population, the researcher has evidence that the treatment has had an effect.
- On the other hand, if the sample is not noticeably different from the original population, it would appear that the treatment has no effect.
- Notice that the interpretation of the research results depends on whether the sample is **noticeably different from** (显著不同于) the population.

Noticeable difference

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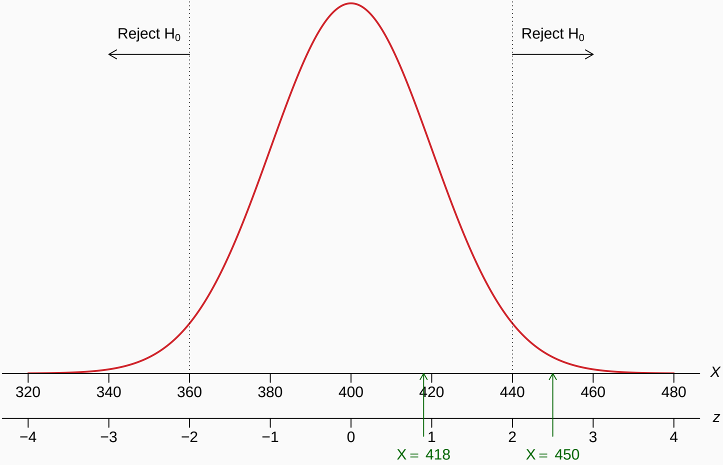
- Thus, we can use z-scores to help decide whether the treatment has caused a change.
- Specifically, if the individuals who receive the treatment finish the research study with extreme z-scores, we can conclude that the treatment does appear to have an effect.

Noticeable difference: An example



Questions?