Chapter 7.
Probability and Samples

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z-score and Probability

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- Whenever a score is selected from a population, you should be able to compute a z-score that describes exactly where the score is located in the distribution.
- If the population is normal, you also should be able to determine the probability value for obtaining any individual score.

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- However, the z-scores and probabilities that we have considered so far are limited to situations in which the sample consists of a single score.
- In this chapter we extend the concepts of z-scores and probability to cover situations with larger samples.
- In particular, we introduce a procedure for transforming a sample mean into a z-score.

Sampling error

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Sampling error

- **Sampling error** (抽样误差) is the natural discrepancy, or amount of error, between a sample statistic and its corresponding population parameter.
- As noted, two separate samples probably will be different even though they are taken from the same population. The samples will have different individuals, different scores, different means, and so on.
- In most cases, it is possible to obtain thousands of different samples from one population.

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- The ability to predict sample characteristics is based on the distribution of sample means.

The Distribution of Sample Means

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- Because statistics are obtained from samples, a distribution of statistics is referred to as a sampling distribution.

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- **A sampling distribution** (样本分布) is a distribution of statistics (统计量) obtained by selecting all the possible samples of a specific size from a population.
- The distribution of sample means is an example of a sampling distribution.
- **The distribution of sample means** (样本平均值的分布) is the collection of sample means for all the possible random samples of a particular size (n) that can be obtained from a population.

Characteristics of the Distribution of Sample Means

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- Samples are not expected to be perfect but they are representative of the population.
- As a result, most of the sample means should be relatively close to the population mean.

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- As a result, the sample means should pile up in the center of the distribution (around μ) and the frequencies should taper off as the distance between M and μ increases.
- This describes a normal-shaped distribution.

Characteristics of the Distribution of Sample Means

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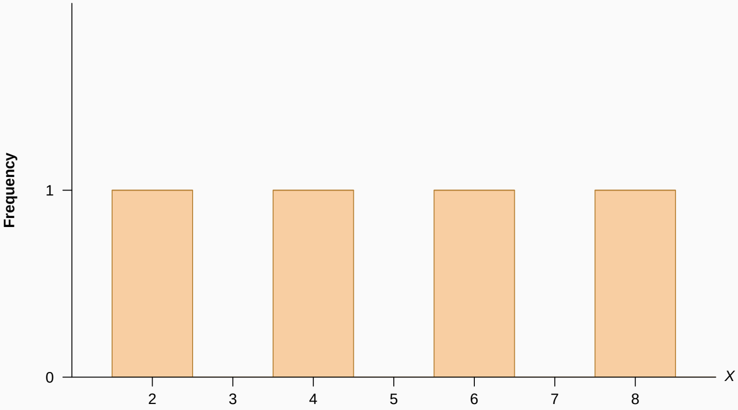
- Logically, a large sample should be a better representative than a small sample.
- Thus, the sample means obtained with a large sample size should cluster relatively close to the population mean; the means obtained from small samples should be more widely scattered.

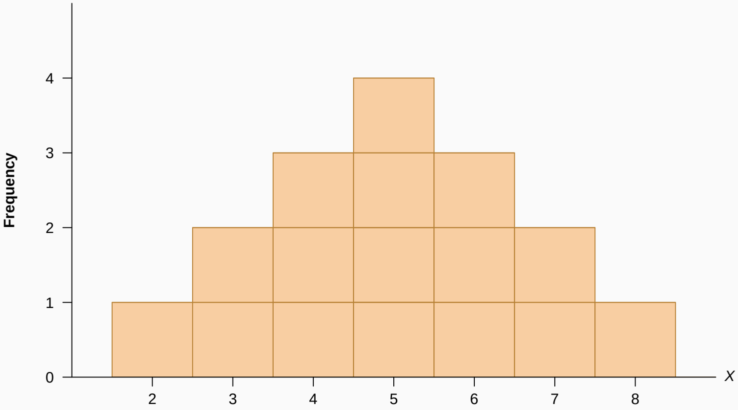
Characteristics of the Distribution of Sample Means



Characteristics of the Distribution of Sample Means

	S1	S2	Mean
1	2	2	2.00
2	4	2	3.00
3	6	2	4.00
4	8	2	5.00
5	2	4	3.00
6	4	4	4.00
7	6	4	5.00
8	8	4	6.00
9	2	6	4.00
10	4	6	5.00
11	6	6	6.00
12	8	6	7.00
13	2	8	5.00
14	4	8	6.00
15	6	8	7.00
16	8	8	8.00

Characteristics of the Distribution of Sample Means



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$$p(M > 7) = \frac{1}{16}$$

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- The Distribution of Sample Means is almost perfectly normal if either of the following two conditions is satisfied:

1. The population from which the samples are selected is a normal distribution.
2. The number of scores (n) in each sample is relatively large, around 30 or more.

The Expected Value of M

The Expected Value of M

- The mean of the distribution of sample means is equal to the mean of the population of scores, μ, and is called the **expected value of M** (样本平均值的期望值).

The Standard Error of M

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- The standard deviation of the distribution of sample means, σ_M, is called the **standard error of M** (样本平均值的标准误).
- The standard error provides a measure of how much distance is expected on average between a sample mean (M) and the population mean (μ).

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- The standard error describes the distribution of sample means.
- Standard error measures how well an individual sample mean represents the entire distribution.

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1. The size of the sample and
2. The standard deviation of the population from which the sample is selected.

The Standard Error of M

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- The **law of large numbers** (大数定理) states that the larger the sample size (n), the more probable it is that the sample mean will be close to the population mean.

The Standard Error of M

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- The formula for standard error expresses the relationship between standard deviation and sample size (n).

$$\text{standard error} = \sigma_M = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sigma^2}{n}} \tag{1}$$

The Standard Error of M

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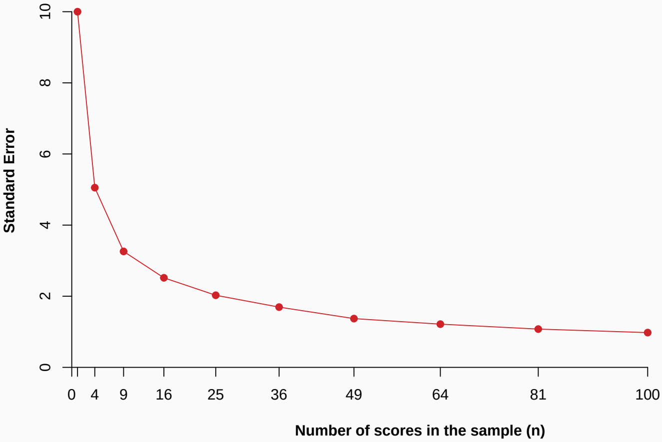
1. When the sample consists of a single score ($n = 1$), the standard error is the same as the standard deviation ($\sigma_M = \sigma$).
2. As sample size (n) increases, the size of the standard error decreases. (Larger samples are more accurate.)

The Standard Error of M

	Sample_Size	Standard_Error
1	1	10.00
2	4	5.06
3	9	3.36
4	16	2.45
5	25	2.01
6	36	1.66
7	49	1.48
8	64	1.25
9	81	1.16
10	100	1.02

The Standard Error of M



Central Limit Theorem

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- For any population with mean μ and standard deviation σ, the distribution of sample means for sample size n will have a mean of μ and a standard deviation of $\sigma/\sqrt{n}$ and will approach a normal distribution as n approaches infinity.

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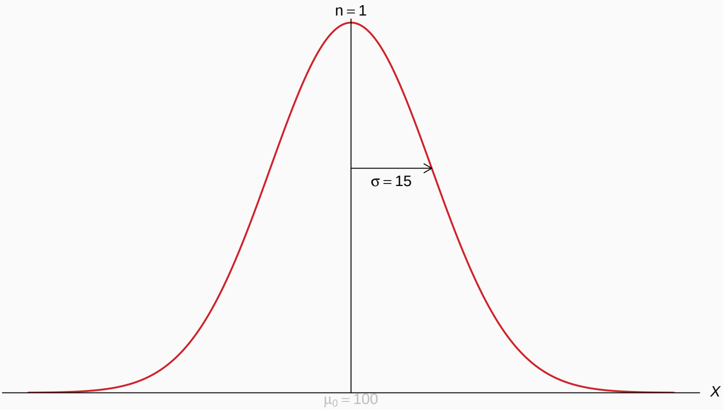
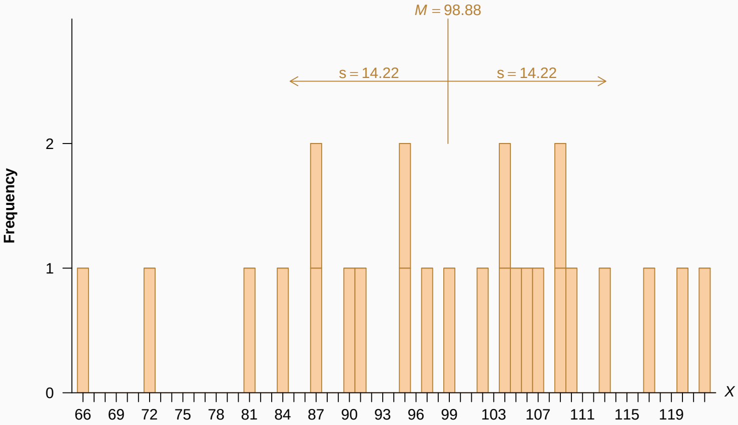
Central Limit Theorem

- The value of this theorem comes from two simple facts:
- First, it describes the distribution of sample means for *any population*, no matter what shape, mean, or standard deviation.
- Second, the distribution of sample means "approaches" a normal distribution *very rapidly*.
- By the time the sample size reaches $n = 30$, the distribution is almost perfectly normal.

Three Different Distributions



Three Different Distributions



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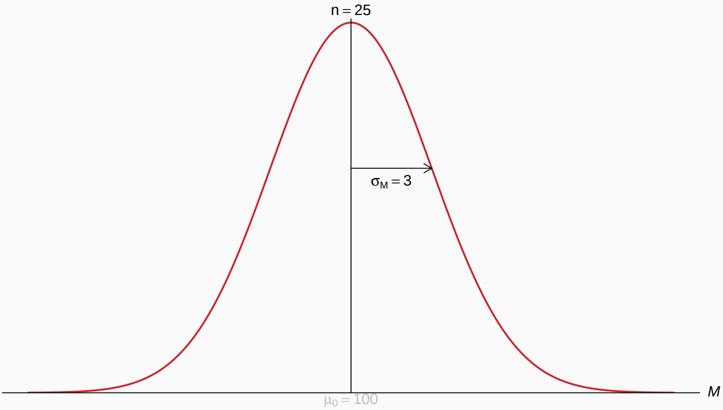

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Probability and the Distribution of Sample Means

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- The population of scores on the SAT forms a normal distribution with $\mu = 500$ and $\sigma = 100$.

```
mu <- 500
sigma <- 100
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- If you take a random sample of $n = 16$ students, what is the probability that the sample mean will be greater than $M = 525$?

```
n <- 16
M <- 525
```

Probability and the Distribution of Sample Means

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- If you take a random sample of $n = 16$ students, what is the probability that the sample mean will be greater than $M = 525$?

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n <- 16
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```

- The distribution of sample means has the following characteristics:

Probability and the Distribution of Sample Means

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- The distribution has a mean of 500 because the population mean is $\mu = 500$.

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mu_m <- mu
```

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- The distribution has a mean of 500 because the population mean is $\mu = 500$.

```
mu_m <- mu
```

- For $n = 16$, the distribution has a standard error of:

```
(sigma_m <- sigma / sqrt(n))
## [1] 25
```

Probability and the Distribution of Sample Means

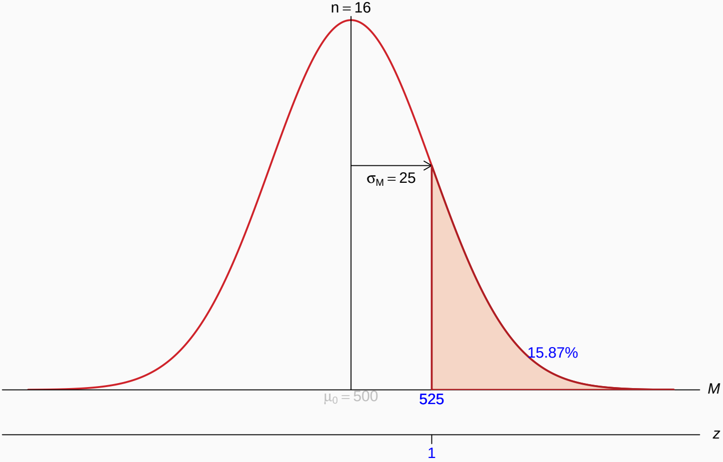
Probability and the Distribution of Sample Means

- The probability that the sample mean will be greater than $M = 525$ is

```
pnorm(q = 525, mean = mu_m, sd = sigma_m,
      lower.tail = FALSE)
```

```
## [1] 0.1586553
```

Probability and the Distribution of Sample Means



A z-Score for Sample Means

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- the z-score formula for locating a sample mean is

$$z = \frac{M - \mu}{\sigma_M} \tag{2}$$

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$$z = \frac{M - \mu}{\sigma_M} \tag{2}$$

- Just as every score (X) has a z-score that describes its position in the distribution of scores, every sample mean (M) has a z-score that describes its position in the distribution of sample means.

A z-Score for Sample Means

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- Specifically, the sign of the z-scores tells whether the sample mean is above (+) or below (-) μ and the numerical value of the z-score is the distance between the sample mean and μ in terms of the number of standard errors.

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- Specifically, the sign of the z-scores tells whether the sample mean is above (+) or below (-) μ and the numerical value of the z-score is the distance between the sample mean and μ in terms of the number of standard errors.
- When the distribution of sample means is normal, it is possible to use z-scores and the unit normal table to find the probability associated with any specific sample mean.

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- Once again, the distribution of SAT scores forms a normal distribution with a mean of $\mu = 500$ and a standard deviation of $\sigma = 100$.

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- For this example, we are going to determine what kind of sample mean is likely to be obtained as the average SAT score for a random sample of $n = 25$ students.

A z-Score for Sample Means

- Once again, the distribution of SAT scores forms a normal distribution with a mean of $\mu = 500$ and a standard deviation of $\sigma = 100$.
- For this example, we are going to determine what kind of sample mean is likely to be obtained as the average SAT score for a random sample of $n = 25$ students.
- Specifically, we will determine the exact range of values that is expected for the sample mean 80% of the time.

A z-Score for Sample Means

```
min <- qnorm(p = 0.5 - 0.8 / 2, mean = 500,
    sd = 100 / sqrt(25), lower.tail = TRUE)
(min <- round(min, digits = 1))

## [1] 474.4
```

A z-Score for Sample Means

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min <- qnorm(p = 0.5 - 0.8 / 2, mean = 500,
    sd = 100 / sqrt(25), lower.tail = TRUE)
(min <- round(min, digits = 1))

## [1] 474.4
```
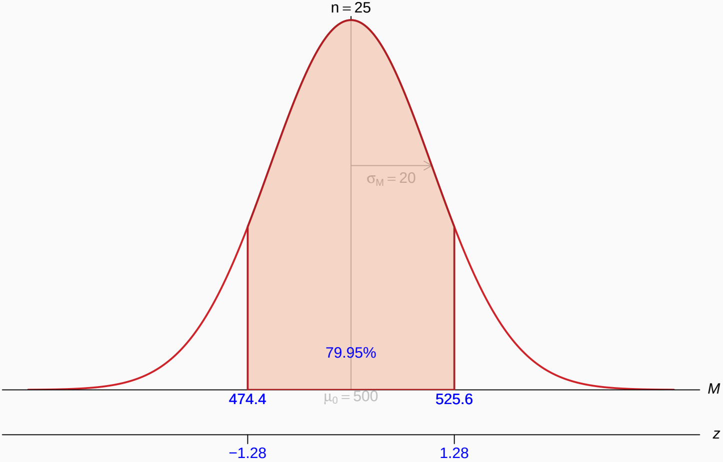

```
max <- qnorm(p = 0.5 + 0.8 / 2, mean = 500,
    sd = 100 / sqrt(25), lower.tail = TRUE)
(max <- round(max, digits = 1))

## [1] 525.6
```

A z-Score for Sample Means

```
c(min, max)

## [1] 474.4 525.6
```

A z-Score for Sample Means



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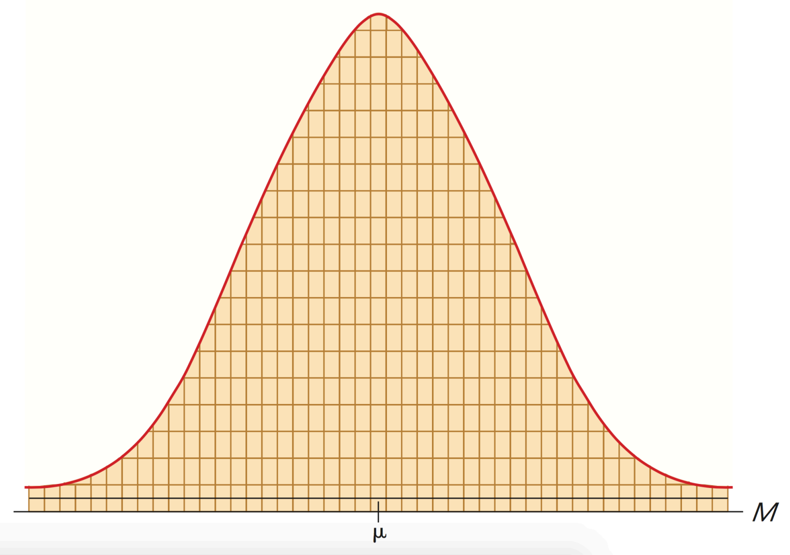
More about Standard Error

- Two adjustments:
- We are now using the distribution of sample means instead of a distribution of scores.
- We are now using the standard error instead of the standard deviation.
- One single rule:
- Whenever you are working with a sample mean, you must use the standard error.

A prototypical distribution of sample means



Sampling Error and Standard Error

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- Standard Error. The standard error provides a method for defining and measuring sampling error.
- Knowing the standard error gives researchers a good indication of how accurately their sample data represent the populations they are studying.

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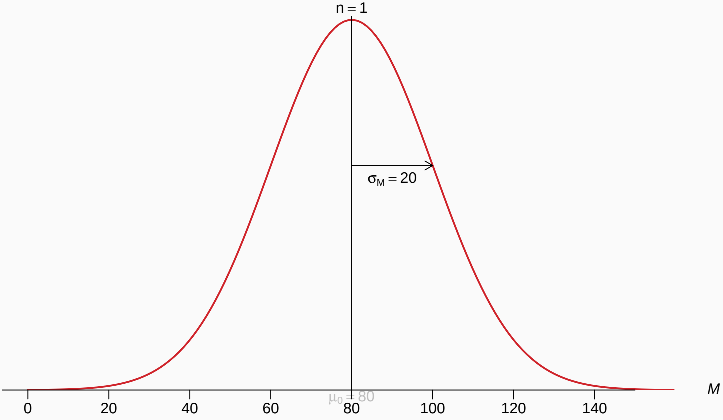
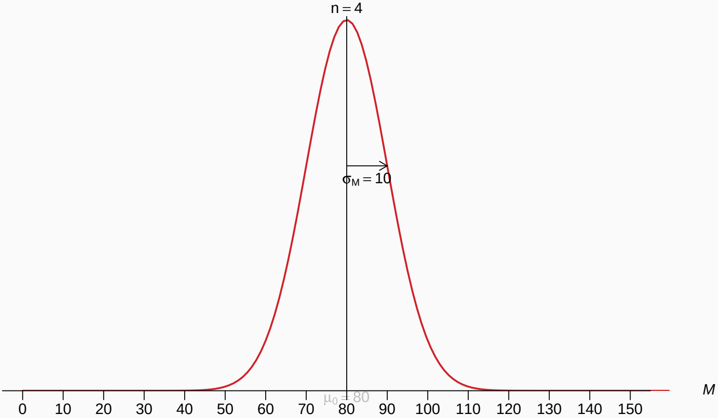
- The average response was $\mu = 80$ minutes, and the distribution of viewing times was approximately normal with a standard deviation of $\sigma = 20$.
- Next, we take a sample from this population and examine how accurately the sample mean represents the population mean.
- More specifically, we will examine how sample size affects accuracy by considering three different samples: one with $n = 1$ student, one with $n = 4$ students, and one with $n = 100$ students.

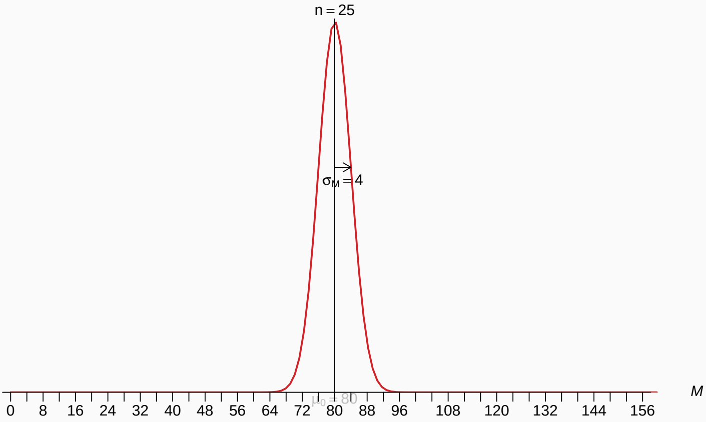
Sampling Error and Standard Error: n = 1



Sampling Error and Standard Error: n = 4



Sampling Error and Standard Error: n = 25



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- As we will see in future chapters, the standard error plays a very important role in inferential statistics.
- Because of its crucial role, the standard error for a sample mean, rather than the sample standard deviation, is often reported in scientific papers.
- Scientific journals vary in how they refer to the standard error, but frequently the symbols SE and SEM (for standard error of the mean) are used.

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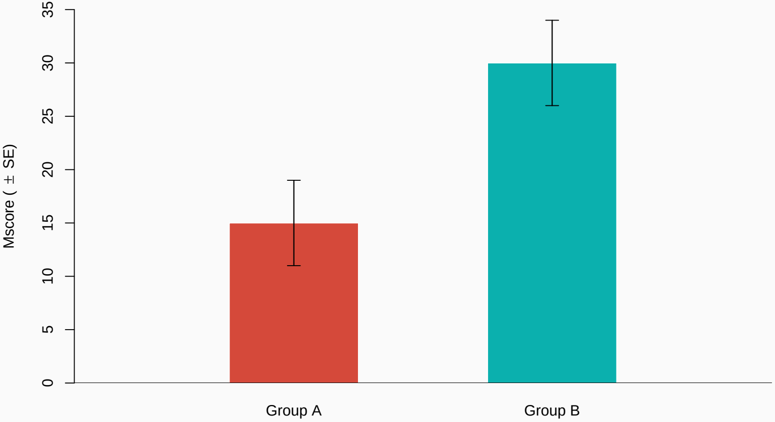
- Much like the standard deviation, it may be reported in a table along with the sample means.
- Alternatively, the standard error may be reported in graphs.

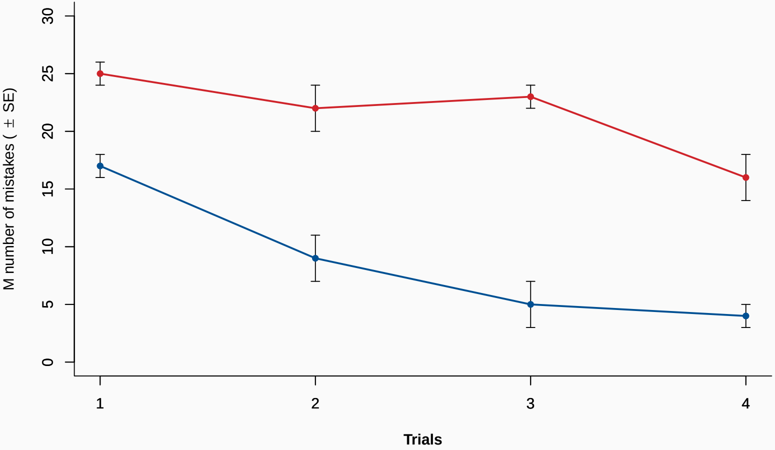
Reporting Standard Error: Table

	n	Mean	SE
Control	17	32.23	2.31
Camera	15	45.17	2.78

Reporting Standard Error: Bar graph



Reporting Standard Error: Line Graph



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Looking Ahead to Inferential Statistics

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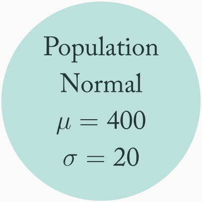
Looking Ahead to Inferential Statistics

- Suppose that a psychologist is planning a research study to evaluate the effect of a new growth hormone.
- It is known that regular, adult rats (with no hormone) weigh an average of $\mu = 400g$.
- Of course, not all rats are the same size, and the distribution of their weights is normal with $\sigma = 20$.
- The psychologist plans to select a sample of $n = 25$ newborn rats, inject them with the hormone, and then measure their weights when they become adults.

Looking Ahead to Inferential Statistics



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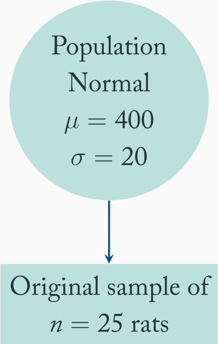
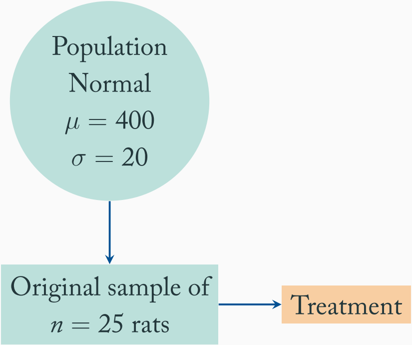


Looking Ahead to Inferential Statistics



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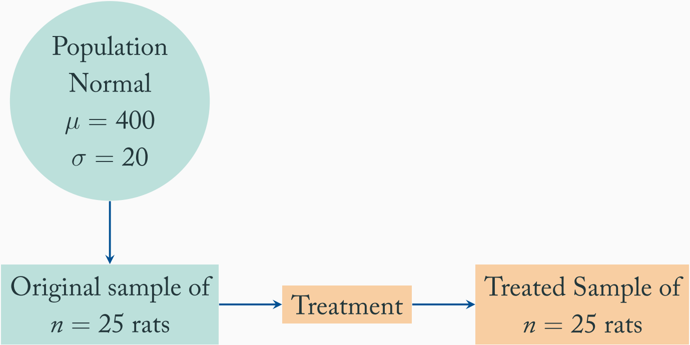

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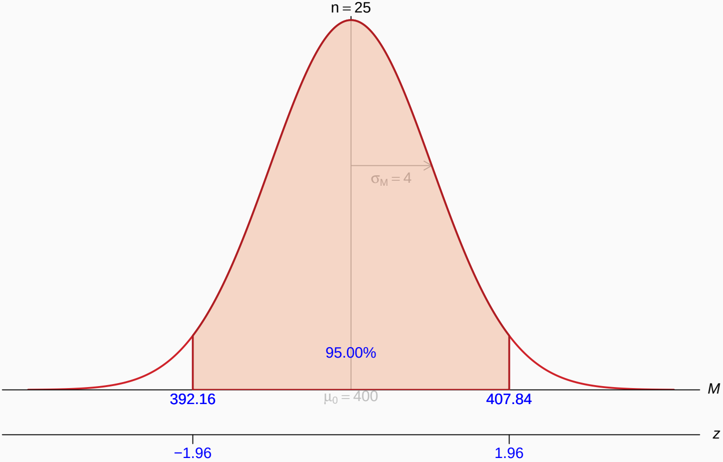

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- If our sample produces one of these extreme means, then we have evidence of a treatment effect.
- Specifically, it is very unlikely that such an extreme outcome would occur without a treatment effect.
- We evaluated the effect of a treatment by determining whether the treated sample was noticeably different from an untreated sample.

Standard Error as a Measure of Reliability

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- The term reliability refers to the consistency of different measurements of the same thing.

Questions?